B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Physics

(Advanced Mathematical Physics)

Paper : DSE-1(1)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

SECTION-I

1. Answer *any five* questions out of *eight* questions carrying 02 marks each. 2×5=10

(a) Show that the vectors (1, *i*) and (1, –*i*) are orthogonal to each other.

(b) Consider real space R^3. The following vectors form a basis set of R^3. $u_1 = (1, -1, 0)$; $u_2 = (1, 1, 0)$ and $u_3 = (0, 1, 1)$. Write down the vector $v = (5, 3, 4)$ in terms of the basis set.

(c) Show that any matrix can be written in terms of a symmetric and antisymmetric matrix.

(d) Express the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{pmatrix}$ as a sum of a lower triangular matrix with zero leading diagonals and an upper triangular matrix.

(e) Let λ_i and μ^i be the components of a covariant and contra-variant vector respectively. Prove that $\lambda_i \mu^i$ is an invariant.

(f) Write down the Quotient law of Tensors. Using this law show that δ^i_j is a mixed tensor of type (1, 1).

(g) If the relation $\lambda^i_j \; A^j = \sigma A^i$ holds for any contravariant vector A^i, where σ is a scalar, show that $\lambda^i_j = \sigma \delta^i_j$.

(h) If $A = \lambda_i x^i$ for all values of the independent variables $x^1, x^2, x^3, \ldots, x^n$ and λ_i's are constants, show that $\frac{\partial}{\partial x^j}\left(\lambda_i x^i\right) = \lambda_j$.

SECTION-II

[Answer *any two* questions out of *four* questions carrying 05 marks each.] 5×2=10

2. Fourier series represents a choice of basis for smooth function on an interval O to L. One such choice of basis is $\hat{e}_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$; consider n is a positive integer. Show that the basis set is both mutually orthogonal and normalised set of basis vectors. [Use scalar product identity to prove this]

3. Find the eigenvalues and normalised eigenvectors of the matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. 2+3=5

4. Let us define the line element $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$ in a vector space V_4.
   (a) Write down the matrix form of metric tensor.
   (b) Show that vector $\left(-1, -1, 1, \frac{\sqrt{3}}{c}\right)$ is a null vector. 3+2=5

5. Prove that $\left\{ {i \atop ij} \right\} = \frac{\partial}{\partial x^j}\left(log\sqrt{g}\right)$,
   here $g = g_{ik}\ G_{ik}$ and G_{ik} is the cofactor of g_{ik} in g. And $g = |g_{ik}| > 0$ g_{ik} is fundamental tensor.

SECTION-III

[Answer *any two* questions out of *four* questions carrying 10 marks each.] 10×2=20

6. (a) Verify the Caley-Hamiltonian theorem for the matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$. Hence find A^{-1}.
   (b) Find the normalised eigenvectors of the matrix = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.
   Construct a diagonalising matrix P using the eigenvectors found above. Then verify that P matrix diagonalises A matrix. (3+1)+(3+3)=10

7. (a) Let $B = P^{-1}AP$ where P is a unitary matrix. Show that if A is Hermitian then B is also a Hermitian matrix.
   (b) If P is a non-singular matrix and $P^{-1}AP$ and $P^{-1}BP$ both are diagonal, prove that A and B commute with each other.
   (c) Determine e^A when $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. 2+2+6=10

8. (a) Define orthogonality conditions of two covariant tensors of rank one. Explain all the symbols you use.
   (b) Define angle θ between two non-null contravariant vectors. Explain all the symbols you use.
   (c) If a vector has components $\dot{x}$ and $\dot{y}$ in Cartesian coordinates, then using tensor calculus, show that in polar coordinates the components are $\dot{r}$ and $\dot{\theta}$. Here dot represents differentiation with respect to time. Symbols have their usual meaning. 2+2+6=10

9. (a) Write down the Christoffel 3 index symbol of first kind denoted by $[ij, k]$.
   (b) Write down the Christoffel 3 index symbol of second kind denoted by $\left\{ {m \atop p\ q} \right\}$.
   (c) $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta(d\phi)^2$ defines the line element in a spherical polar coordinates. Find the values of $[33, 2]$ and $\left\{ {1 \atop 22} \right\}$. 1+1+8=10